Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International GCSE

Time 2 hours

Paper reference **4PM1/02R**

Further Pure Mathematics
PAPER 2R

Calculators may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You must **NOT** write anything on the formulae page.
  Anything you write on the formulae page will gain NO credit.

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over ▶

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Pearson

International GCSE in Further Pure Mathematics Formulae sheet

Mensuration

Surface area of sphere = $4\pi r^2$

Curved surface area of cone = $\pi r \times$ slant height

Volume of sphere = $\frac{4}{3}\pi r^3$

Series

Arithmetic series

Sum to n terms, $S_n = \frac{n}{2}[2a + (n-1)d]$

Geometric series

Sum to n terms, $S_n = \frac{a(1-r^n)}{(1-r)}$

Sum to infinity, $S_\infty = \frac{a}{1-r} \quad |r| < 1$

Binomial series

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \quad \text{for} \quad |x| < 1, n \in \mathbb{Q}$$

Calculus

Quotient rule (differentiation)

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mathrm{f}(x)}{\mathrm{g}(x)}\right) = \frac{\mathrm{f}'(x)\mathrm{g}(x) - \mathrm{f}(x)\mathrm{g}'(x)}{[\mathrm{g}(x)]^2}$$

Trigonometry

Cosine rule
In triangle ABC: $a^2 = b^2 + c^2 - 2bc\cos A$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\sin(A+B) = \sin A \cos B + \cos A \sin B$ $\quad$ $\sin(A-B) = \sin A \cos B - \cos A \sin B$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$ $\quad$ $\cos(A-B) = \cos A \cos B + \sin A \sin B$

$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$ $\quad$ $\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$

Logarithms

$\log_a x = \frac{\log_b x}{\log_b a}$

Answer all ELEVEN questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 Find the set of values for x for which

(a) $5x - 10 > 4x - 7$

(1)

(b) $2x^2 - 11x + 5 < 0$

(3)

(c) **both** $5x - 10 > 4x - 7$ **and** $2x^2 - 11x + 5 < 0$

(1)

(Total for Question 1 is 5 marks)

2 The point A has coordinates $(-7, -1)$ and the point B has coordinates $(3, 4)$

(a) Find an equation of the line that passes through A and B
Give your answer in the form $ax + by + c = 0$ where a, b and c are integers.

(3)

The point C has coordinates $(-3, 7)$

Given that k is a constant such that $AB = kAC$

(b) find the value of k

(2)

The point D has coordinates $(3, p)$ where p is a constant.

Given that CD is perpendicular to AB

(c) find the value of p

(3)

Question 2 continued

(Total for Question 2 is 8 marks)

3 Differentiate with respect to x

(a) $e^{2x}\sqrt{5x-3}$

(3)

(b) $\dfrac{x^3}{\cos 3x}$

(3)

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Question 3 continued

(Total for Question 3 is 6 marks)

4 The quadratic equation

$$2x^2 + 4x + 3 = 0$$

has roots α and β

(a) Without solving the equation, show that $\alpha^2 + \beta^2 = 1$

(4)

(b) Without solving the equation, find the value of $\alpha^4 + \beta^4$

(3)

(c) Hence form a quadratic equation with integer coefficients that has roots α^4 and β^4

(3)

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Question 4 continued

Question 4 continued

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Question 4 continued

(Total for Question 4 is 10 marks)

5 A geometric series G has first term 12 and common ratio $\frac{3}{8}$

(a) Find the sum to infinity of G

(2)

(b) Show that the 6th term of G can be written as $\frac{3^6}{2^{13}}$

(3)

The nth term of G is u_n

(c) By finding an expression for u_n in terms of n, show that

$$\log_2 u_n = n\log_2 3 - 3n + 5$$

(5)

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Question 5 continued

Question 5 continued

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Question 5 continued

(Total for Question 5 is 10 marks)

6 The curve C has equation

$$y = 4\sqrt{x}$$

The point A on C has coordinates (9, 12)

The tangent to C at the point A meets the x-axis at the point T

(a) Find the coordinates of T

(5)

The normal to C at the point A meets the x-axis at the point N

(b) Find the coordinates of N

(4)

(c) Calculate the area of triangle ATN

(2)

Question 6 continued

(Total for Question 6 is 11 marks)

7 Given that

$$\frac{3 + \sin^2\theta}{\cos\theta - 2} = 3\cos\theta$$

(a) show that $\cos\theta = -\frac{1}{2}$

(4)

(b) Hence solve the equation

$$\frac{3 + \sin^2 3x}{\cos 3x - 2} = 3\cos 3x \quad \text{for } 0° \leqslant x < 180°$$

(4)

Question 7 continued

(Total for Question 7 is 8 marks)

8 Liquid drips onto a large horizontal flat cloth, forming a circular stain.

The liquid starts to drip onto the cloth at time $t = 0$

The area of the stain increases at a constant rate of $1.5\,\text{cm}^2/\text{s}$

(a) Find, in terms of π, the radius of the stain at time $t = 4$ seconds.

(3)

(b) Find, in cm/s to 3 significant figures, the rate at which the radius of the stain is increasing at time $t = 4$ seconds.

(4)

Question 8 continued

(Total for Question 8 is 7 marks)

9 Given that α is the acute angle such that $\tan\alpha = \frac{2}{3}$

(a) find the exact value of $\cos\alpha$

(1)

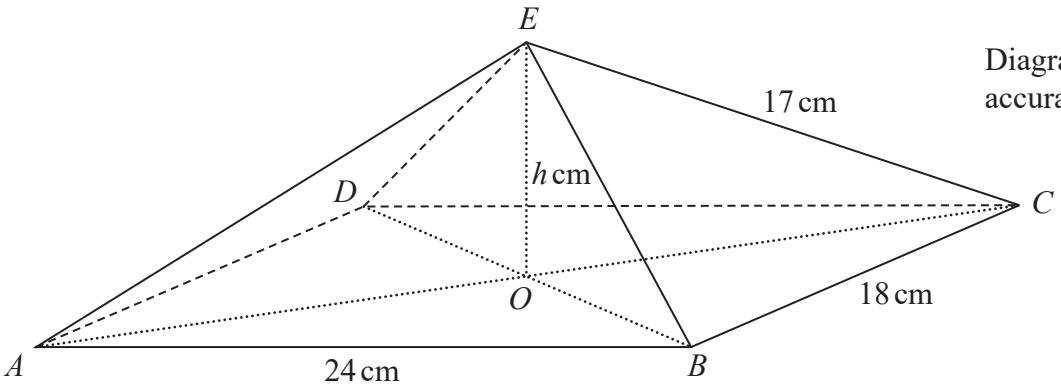

Diagram **NOT** accurately drawn

Figure 1

Figure 1 shows a right pyramid with a rectangular base $ABCD$ and vertex E

The rectangular base of the pyramid is horizontal with $AB = 24$ cm and $BC = 18$ cm.

The diagonals of the base intersect at the point O

The vertex E of the pyramid is vertically above O such that

$$AE = BE = CE = DE = 17 \text{ cm}$$

The height of the pyramid is h cm.

(b) Find the value of h

(3)

The size of the angle between the plane EBC and the plane $ABCD$ is $\theta°$

(c) Show that $\tan\theta° = \frac{2}{3}$

(2)

The point P is the midpoint of EB and the point Q is the midpoint of EC

(d) Find the size, in degrees to one decimal place, of the angle between the plane OPQ and the plane $BCQP$

(4)

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Question 9 continued

Question 9 continued

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Question 9 continued

(Total for Question 9 is 10 marks)

10

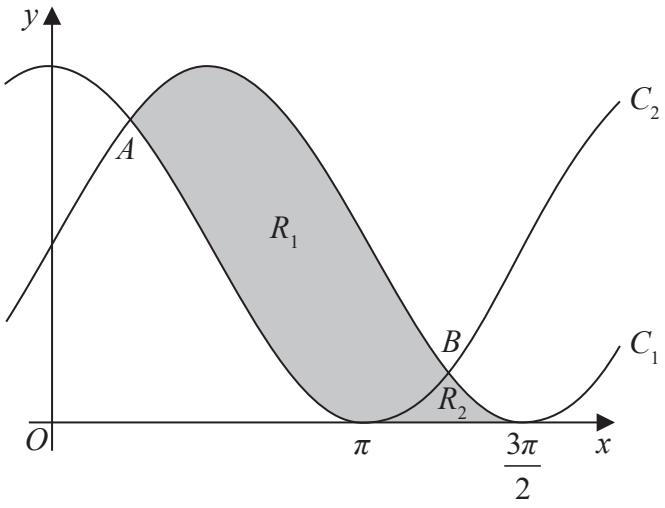

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Figure 2

Figure 2 shows part of the curve C_1 with equation $y = \sin x + 1$ and part of the curve C_2 with equation $y = \cos x + 1$

As shown in Figure 2, C_1 and C_2 intersect at the point A and at the point B

(a) Find the exact value of the x coordinate of A and the exact value of the x coordinate of B

(3)

The shaded finite region R_1 shown in Figure 2 is bounded by C_1 and C_2

The shaded finite region R_2 shown in Figure 2 is bounded by the x-axis, C_1 and C_2

(b) Use calculus to find the ratio

$$\text{area of } R_1 : \text{area of } R_2$$

Give your answer in the form $a : \left(\frac{\pi\sqrt{2}}{b} - c \right)$ where a, b and c are integers.

(9)

Question 10 continued

Question 10 continued

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Question 10 continued

(Total for Question 10 is 12 marks)

11

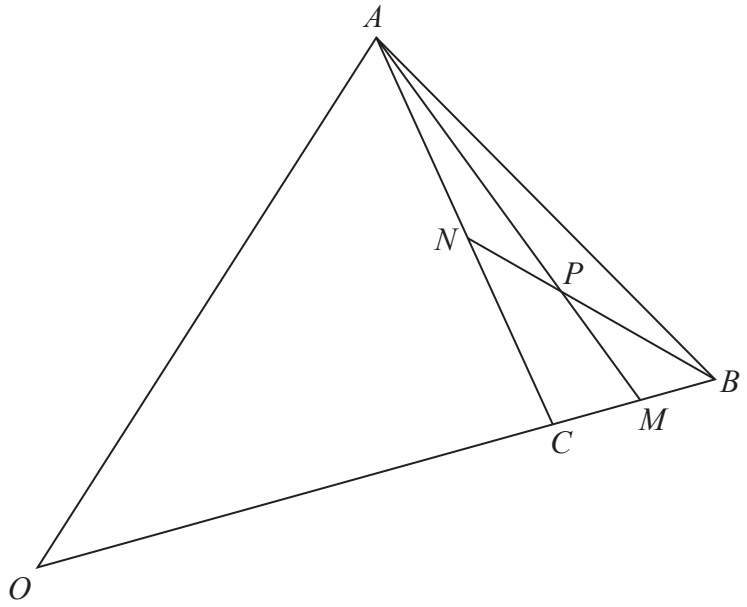

Diagram **NOT** accurately drawn

Figure 3

Figure 3 shows triangle OAB with $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$

The point C lies on OB such that $OC : CB = 2 : 1$
The point M is the midpoint of CB and the point N is the midpoint of AC
The lines AM and NB intersect at the point P

(a) Using a vector method, find $\overrightarrow{OP}$ as a simplified expression in terms of **a** and **b**

(9)

The point Q is the midpoint of AB

(b) Using a vector method, show that C, P and Q are collinear.

(4)

Question 11 continued

Question 11 continued

(Total for Question 11 is 13 marks)

TOTAL FOR PAPER IS 100 MARKS

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